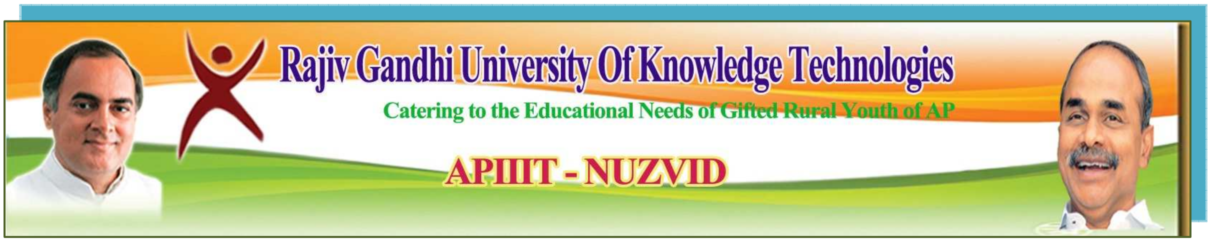


MATHEMATICS _ V SEMISTER

Like the crest of a peacock so is mathematics at the head of all knowledge.

UNIT:I

PARAMETERIZATIONS:-

1. The parametric equations:-

   A. Line:

      a. Line segment: $x=x_1+(x_2-x_1)t$ and $y=y_1+(y_2-y_1)t$ and $0\le t\le 1$

      b. Line: $x=x_1+(x_2-x_1)t$ and $y=y_1+(y_2-y_1)t$ and $-\infty<t<\infty$

      c. Ray : $x=x_1+(x_2-x_1)t$ and $y=y_1+(y_2-y_1)t$ and $t\ge 0$

   B. Circle: $x^2+y^2=a^2$

      a. $x = a\cos nt$ and $y = a\sin nt$ and $0 \le t \le \frac{2\pi m}{n}$ counterclockwise

         $x = a\cos nt$ and $y = -a\sin nt$ and $0 \le t \le \frac{2\pi m}{n}$ clockwise direction

         m=Number of times traces the circle

   C. Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

      a. $x = a\cos nt$ and $y = b\sin nt$ and $0 \le t \le \frac{2\pi m}{n}$ counterclockwise

      b. $x = a\cos nt$ and $y = -b\sin nt$ and $0 \le t \le \frac{2\pi m}{n}$ clockwise direction

         m=Number of times traces the circle

   D. Hyperbola:

      $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $\quad x = \pm a\sec t$ and $y = b\tan t$ and $-\frac{\pi}{2} < t < \frac{\pi}{2}$

      $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ $\quad x = a\tan t$ and $y = \pm b\sec t$ and $-\frac{\pi}{2} < t < \frac{\pi}{2}$

Hyperbola	Focal axis	Parametric equations	Hyperbola type

$x^2 - y^2 = 1$	x-axis	$x = \sec t$ and $y = \tan t$ and $-\frac{\pi}{2} < t < \frac{\pi}{2}$	Right half
		$x = -\sec t$ and $y = \tan t$ and $-\frac{\pi}{2} < t < \frac{\pi}{2}$	Left half
$y^2 - x^2 = 1$	y-axis	$x = \tan t$ and $y = \sec t$ and $-\frac{\pi}{2} < t < \frac{\pi}{2}$	Upper half
		$x = \tan t$ and $y = -\sec t$ and $-\frac{\pi}{2} < t < \frac{\pi}{2}$	Lower half

E. Cycloid:

a. $x = a(t - \sin t)$ and $y = a(1 - \cos t)$ and $0 \le t \le 2\pi$

b. Length=8a

c. The area of the surface swept by revolving the cycloid about its base is $\frac{64\pi a^2}{3}$

2. x=a cost and y=a sint , 0≤t≤2π be a circle then
   a. Length of the circle =2πa
   b. The area of the surface swept by revolving the circle about either x-axis or y-axis $=4\pi a^2$
3. The equation of a tangent to the ellipse $x = a\cos t$ and $y = b\sin t$ and $0 \le t \le 2\pi$ at the point $t = \frac{\pi}{4}$ is $bx + ay = \sqrt{2}ab$
4. The length of the parameterized curve $x = f(t);\ y = g(t)$ and $a \le t \le b$ is $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
5. The surface area generated by the parametric curve $x = f(t);\ y = g(t)$ and $a \le t \le b$ revolving about
   a. x-axis: **y≥0**

$$S = \int_a^b 2\pi\, y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

   b. y-axis: **x≥0**

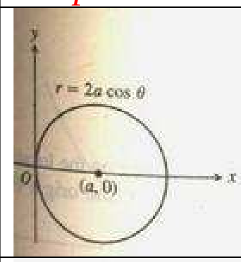
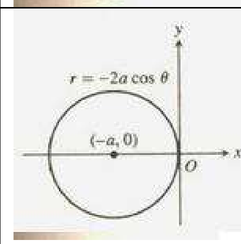
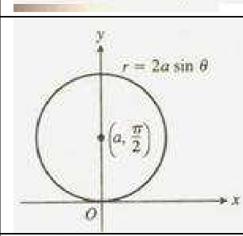
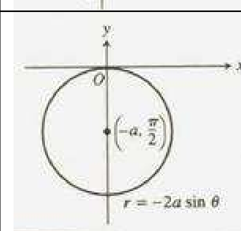
$$S = \int_a^b 2\pi\, x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

UNIT:II

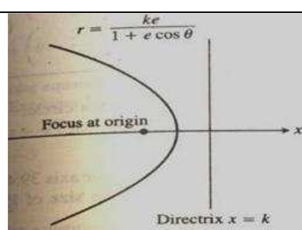
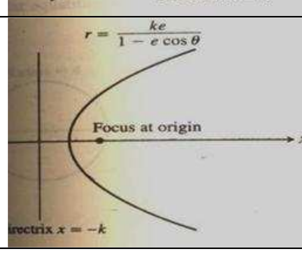
POLAR COORDINATES:

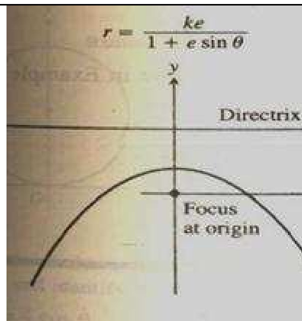
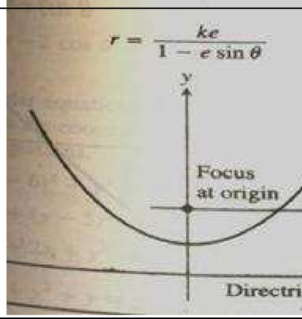
1. The polar coordinates of the Cartesian point (x,y) are (r,θ) where
   $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
2. The Cartesian coordinate of the polar coordinate (r,θ) are (x,y) here
   $x = r\cos\theta$ and $y = r\sin\theta$
3. The equation r=a represents a circle of centre (0,0) and radius a
4. The equation of a straight line passing through origin making an angle θ_0 with the positive x-axis is $\theta = \theta_0$
5. The all polar coordinates of (r , θ) are $(r,\theta) = (r,\theta + 2n\pi) = (-r,\theta + (2n+1)\pi)$ here $n \in Z$
6. Symmetry:-
   a. x-axis:- (r,θ) replaced by $(r,-\theta)$ or $(-r,\pi - \theta)$
   b. y-axis:- (r,θ) replaced by $(-r,-\theta)$ or $(r,\pi - \theta)$
   c. Origin:- (r,θ) replaced by $(-r,\theta)$ or $(r,\pi + \theta)$
7. The slope of a polar curve $r = f(\theta)$ at (r,θ) is $\left(\frac{dy}{dx}\right)_{(r.\theta)} = \frac{f^1(\theta)\sin\theta + f(\theta)\cos\theta}{f^1(\theta)\cos\theta - f(\theta)\sin\theta}$
8. If the curve $r = f(\theta)$ passes through the origin at $\theta = \theta_0$ the slope of the curve at $(0, \theta_0)$ is $\left(\frac{dy}{dx}\right)_{(0.\theta_0)} = \tan\theta_0$
9. If the point (r_0,θ_0) is the foot of perpendicular from origin to the line L then general equation of a straight line is $r_0 = r\cos(\theta - \theta_0)$
10. The general equation of a circle in polar form $a^2 = r_0^{\,2} + r^2 - 2r_0 r\cos(\theta - \theta_0)$
    Here centre=(r_0,θ_0) and radius=a
    The general equation of a circle passing through origin in polar form is $r = 2a\cos(\theta - \theta_0)$ Here centre=(a,θ_0) and radius=a
11. The area of the fan shaped polar curve $r = f(\theta)$,$\alpha \le \theta \le \beta$, is $A = \int_\alpha^\beta \frac{1}{2} r^2 d\theta$
12. The area between two curves $r_1 = f_1(\theta)$ and $r_2 = f_2(\theta)$: $\alpha \le \theta \le \beta$ and $0 \le r_1 \le r_2$
    is $A = \int_\alpha^\beta \frac{1}{2}\left(r_2^{\,2} - r_1^{\,2}\right) d\theta$

13. Different form of circles:

Circle	Centre	Radius	Centre lies on	Graph
$r=2a\cos\theta$	$(a,0)$	a	Positive x-axis	
$r=-2a\cos\theta$	$(-a,0)$	a	Negative x-axis	
$r=2a\sin\theta$	$(a, \pi/2)$	a	Positive y-axis	
$r=-2a\sin\theta$	$(-a,\pi/2)$	a	Negative y-axis	

14. Polar equations for a conic:-

Conic	Graphs	Directrix	Conic is a parabola	Conic is a ellipse
$r=\dfrac{ke}{1+e\cos\theta}$		$x=k$	Vertex= $(r_1,0)$ or $(k/2,0)$	Vertices= $(r_1,0)$ and (r_2,π) or $(a-ae,0)$ and $(a+ae,\pi)$ Centre= (ae,π)
$r=\dfrac{ke}{1-e\cos\theta}$		$x=-k$	Vertex= $(r_1,0)$ or $(k/2,\pi)$	Vertices= $(r_1,0)$ and (r_2,π) or $(a+ae,0)$ and $(a-ae,\pi)$ Centre= $(ae,0)$

$r=\dfrac{ke}{1+e\sin\theta}$		$y=k$	Vertex=**(r_1,π/2)** or (k/2,π/2)	Vertices=**(r_1,π/2a** **and (r_2,π/2)** or $\left(a-ae,\dfrac{\pi}{2}\right)$ and $\left(a+ae,-\dfrac{\pi}{2}\right)$ Centre=**(ae,-π/2)**
$r=\dfrac{ke}{1-e\sin\theta}$		$y=-k$	Vertex=**(r_1,-π/2)** or (k/2,-π/2)	Vertices=**(r_1,π/2)** **and (r_2,-π/2)** or $\left(a+ae,\dfrac{\pi}{2}\right)$ and $\left(a-ae,-\dfrac{\pi}{2}\right)$ Centre=**(ae,π/2)**

Note: In the above table $a=\dfrac{ke}{1-e^2}$

15. *The value of eccentricity:*
    a. *e=1 parabola*
    b. *e>1 hyperbola*
    c. *0<e<1 ellipse*
    d. *e=0 circle*
16. *Distance between centre of the ellipse to directrix= a/e*
17. *Distance between centre of the ellipse to foci= ae*
18. *Distance between centre of the ellipse to vertex =a*
19. *The length of latus rectum of a ellipse or a Hyperbola =2ke*
20. *Polar equation of a ellipse with eccentricity e and semi major axis a*
    $r=\dfrac{a(1-e^2)}{1+e\cos\theta}$
21. *The area of the any cardioid* $r=a(1\pm\cos\theta)$ *or* $r=a(1\pm\sin\theta)$ *is* $\dfrac{3\pi a^2}{2}$ *sq. units*
22. *The area of the shaded region shaded by the circles r=2acosθ and r=2asinθ is* $a^2\left(\dfrac{\pi}{2}-1\right)$
23. *Different types of Limacons:* $r=a\pm b\cos\theta$ *or* $r=a\pm b\sin\theta$; $a\geq 0; b\geq 0$

a/b value	Nature of the graph
Less than 1	Cardioid with inner loop
Equal to 1	cardioid
Lies between 1 and 2	Dimpled limacon
Greater than 2	Oval or Convex limacon

24.The length of the polar curve : $r = f(\theta)$ $,\alpha \le \theta \le \beta$ is $L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

25.The length of the cardioid $r=a(1\pm\cos\theta)$ or $r=a(1\pm\sin\theta)$ is $8a$

26.The surface area generated by the parametric curve $r = f(\theta)$ $,\alpha \le \theta \le \beta$ revolving about

a. x-axis: $\boldsymbol{y \ge 0}$

$$L = \int_{\alpha}^{\beta} 2\pi\, r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

b. y-axis: $\boldsymbol{x \ge 0}$

$$L = \int_{\alpha}^{\beta} 2\pi\, r \cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

27. Length of the circle $r=a\cos\theta$ is πa

28.The general form of equation of leminiscate $\boldsymbol{r^2=a\cos b\theta}$ or $\boldsymbol{r^2=a\sin b\theta}$

29.The area of the surface generated by revolving the Lemniscate $r^2=\cos2\theta$ about

a. x-axis: $2\pi(2-\sqrt{2})$

b. y-axis: $2\sqrt{2}\pi$

30. The length of the curve $r = \sqrt{1+\cos 2\theta}$ and $r = \sqrt{1+\sin 2\theta}$ and $0 \le \theta \le \pi\sqrt{2}$ is 2π

31.Rose curves: general polar equation for rose curves is $r=a\cos b\theta$ or $r=a\sin b\theta$

a. b petals if b is odd

b. 2b petals if b is even.

32.The distance between two polar coordinates $(r_1,\theta_1)$and(r_2,θ_2) is

$$d = \sqrt{r_1^{\,2} + r_2^{\,2} - 2r_1r_2\cos(\theta_1-\theta_2)}$$

UNIT:III

DIFFERENTIAL EQUATIONS:

1. *The problem of finding a function y of x when we know its derivative and its value y_o at particular point x_o is called first order initial value problem(IVP)*
   General form is $\frac{dy}{dx} = f(x,y)\,, y_0 = y(x_0)$
2. *The law of exponential change:* $y = y_0 e^{kt}$ *. If y is a positive and increasing then k is a positive (growth) k>0 and if y is a positive and decreasing then k is negative (decay) k<0*
3. *The number of radioactive elements presents at a time t is given by* $y = y_0 e^{-kt}$ *and k>0*
4. *The half life of radioactive element is given by* $t = \frac{\ln 2}{k}$
5. *The equation for Newton's law of cooling :* $T - T_S = (T_0 - T_S)e^{-kt}$
   T_s*=surrounding temperature*
   T_0*=Initial temperature*
6. *The standard form first order linear differential equation in in y is*
   $\frac{dy}{dx} + P(x)y = Q(x)$ *and* $I.F = e^{\int P(x)dx}$ *and the general solution is*
   $y.(I.F) = \int Q(x)\,(I.F)\;\; dx \;\; + c$
7. *The standard form first order linear differential equation in in x is*
   $\frac{dx}{dy} + P(y)x = Q(y)$ *and* $I.F = e^{\int P(y)dy}$ *and the general solution is*
   $x(I.F) = \int Q(y)\,(I.F)\;\; dy \;\; + c$
8. *The integrating factor of the first order linear differential equation*
   $x\frac{dy}{dx} + ny = Q(x)\;\;, n \in z$ *is I.F=*x^n *(competitive oriented)*
9. *If a body of mass m is coasting to stop and the only force acting on the body is resistance proportional to its velocity v and the distance coasted are given by* $V = V_o e^{-\frac{k}{m}t}$ *and* $D = \frac{V_o m}{k}$

10.The current flowing in the LR-circuit at a time t is given by

$$i=\frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right) \quad or \quad i=I\left(1-e^{-\frac{R}{L}t}\right)$$

11. $I=\frac{V}{R}$ is called the steady state value.

12. $i=\frac{V}{R}-\frac{V}{R}e^{-\frac{R}{L}t}$ here $\frac{V}{R}$ is called steady state solution and $-\frac{V}{R}e^{-\frac{R}{L}t}$ is called transient solution.

13.

t value	$i=I\left(1-e^{-\frac{R}{L}t}\right)$	Percentage in I
$\frac{L}{R}$	0.64I	64%
$\frac{2L}{R}$	0.86I	86%
$\frac{3L}{R}$	0.95I	95%

14.If the switch is thrown open after the current in an RL-circuit has built up to its steady value $I=\frac{V}{R}$,the decay of the current obeys the equation $L\frac{di}{dt}+Ri=0$ then the value of the currents in the following sec are

t value	Current(i)	Percentage in I
$\frac{L}{R}$	$\frac{I}{e}$	36%
$\frac{2L}{R}$	$\frac{I}{e^2}$	14%
$\frac{3L}{R}$	$\frac{I}{e^3}$	5%

UNIT:IV

VECTOR ALGEBRA

1. A vector in the plane determined as the directed line segment.
2. Two vectors are equal if they have the same length and direction.
3. If V is a vector and if V can be written as $V=V_1+V_2$ of two non- parallel vectors V_1 and V_2 then V_1 and V_2 are called the components of V.
4. Two vectors in a plane are said to be parallel if they are nonzero scalar multiples of one another or if the line segments representing them are parallel.
5. The basic vector in the positive direction of X-axis is the vector ***i*** determined by the directed line segment from(0,0,0) to(1,0,0).
6. The basic vector in the positive direction of Y-axis is the vector ***j*** determined by the directed line segment from(0,0,0) to(0,1,0)
7. The basic vector in the positive direction of Z-axis is the vector ***k*** determined by the directed line segment from(0,0,0) to(0,0,1)
8. If V=ai+bj , where a,b are scalars, then the vectors ai and bj are called the vector components of V in the direction of i and j. The numbers a and b are called the scalar components of V in the direction of i and j.
9. The magnitude or length of V=ai+bj is $|V| = \sqrt{a^2+b^2}$
10. The magnitude or length of V=ai+bj+ck is $|V| = \sqrt{a^2+b^2+c^2}$
11. If u is the unit vector obtained by rotating i through an angle θ in the positive direction , then $u = \cos\theta\, i + \sin\theta\, j$. Here cosθ is called the horizontal component of u and sin θ is called vertical component of u.
12. If V is a non zero vector , then
    a. $\frac{V}{|V|}$ is a unit vector in the direction of V,
    b. The equation $V = |V|\left(\frac{V}{|V|}\right)$ express V in terms of its length and direction
13. The slope of the vector V=ai+bj is $\frac{b}{a}$
14. A vector is tangent or normal to a curve at a point if it is parallel or normal to the tangent line that is tangent to the curve at that point.

15. *Planes determined by coordinate axis:*
    a. *Standard equation of xy-plane :* ***z=0***
    b. *Standard equation of yz-plane :* ***x=0***
    c. *Standard equation of xz-plane :* ***y=0***
    d. *The equation of the plane parallel to xy-plane:* ***z=c***
    e. *The equation of the plane parallel to yz-plane:* ***x=a***
    f. *The equation of the plane parallel to zx-plane:* ***y=b***
16. *Three co-ordinate planes x=0, y=0 and z=0 devide the space into eight cells called ocants. In that first octant is* ***{(x,y,z)/x>0,y>0,z>0}***
17. *The equation of the planes perpendicular to x-axis:* ***x=a***
18. *The equation of the planes perpendicular to y-axis:* ***y=b***
19. *The equation of the planes perpendicular to z-axis:* ***z=c***
20. *The equation of the planes parallel to x-axis: The line of intersection of the planes* ***y = b, z = c***
21. *The equation of the planes parallel to y-axis: The line of intersection of the planes* ***z = c,x = a***
22. *The equation of the planes parallel to z-axis: The line of intersection of the planes* ***x = a,y = b***
23. *Position vector: The position vector r from the origin O to the typical point P(a,b,c) is* $r = OP = ai + bj + ck$
24. *The distance between* $P(x_1,y_1,z_1)$ *and* $Q(x_2,y_2,z_2)$ *is*
    $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
25. *The midpoint of the line segments joining the points* $P(x_1,y_1,z_1)$ *and* $Q(x_2,y_2,z_2)$ *is* $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
26. *The unit vector in the direction of the vector from* $P(x_1,y_1,z_1)$ *and* $Q(x_2,y_2,z_2)$ *is* $U = \frac{PQ}{|PQ|}$
27. *The equation of the sphere with centre* $O(x_0,y_0,z_0)$ *with radius a is* $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$
28. *For the Sphere* $x^2 + y^2 + z^2 + 2gx + 2fy + 2hz + c = 0$ *centre =(-g,-f,-h) and radius=*$\sqrt{g^2 + f^2 + h^2 - c}$

29. Dot product or Scalar Product: *The dot product of two vectors A and B is denoted by A.B and is defined as follows* $A\,.\,B = |A||B|\cos\theta$

30. *The scalar product is positive if* θ, *the angle between the vectors is acute and negative if* θ *is obtuse. Note that* $0 \leq \theta \leq \pi$.

31. *If A is a non zero vector then* $|A| = \sqrt{A\,.\,A}$

32. *If* $A = a_1 i + a_2 j + a_3 k$ *and* $B = b_1 i + b_2 j + b_3 k$ *then* $A\,.\,B = a_1 b_1 + a_2 b_2 + a_3 b_3$

33. *The angle between the two non-zero vectors A and B is given by*

$$\theta = \cos^{-1}\left(\frac{A\,.B}{|A||B|}\right) = \cos^{-1}\left(\frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}\,.\sqrt{b_1{}^2 + b_2{}^2 + b_3{}^2}}\right)$$

34. *Two non zero vectors A and B are orthogonal (or perpendicular) if the angle between them is* $\frac{\pi}{2}$

35. *Two non zero vectors A and B are orthogonal iff* $A\,.\,B = 0$

36. Properties of Dot Product:
    a. $A\,.\,B = B\,.\,A$ *(commutative)*
    b. $A.(B + C) = A\,.\,B + A\,.\,C$ *(Left Distributive law)*
    c. $(A + B).\,C = A\,.C + B\,.C$ *(Right Distributive law)*
    d. $(cA)\,.\,B = A\,.(cB) = c\,(A\,.\,B)$

37. *If* θ *is the angle between two unit vectors a and b then*
    a. $|a - b\cos\theta| = \sin\theta$
    b. $\frac{1}{2}|a - b| = \sin\left(\frac{\theta}{2}\right)$

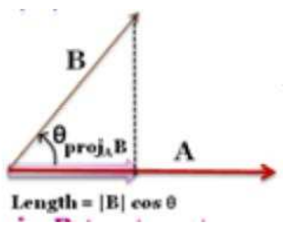
38. *If a, b, c are three non zero vectors such that* $a+b+c=0$ *and* $|a| = x$ *and* $|b| = y$ *and* $|c| = z$ *then* $a.b + b.c + c.a = -\left(\frac{x^2 + y^2 + z^2}{2}\right)$

39. *The vector projection of* $B = P\vec{Q}$ *onto a non zero vector* $A = P\vec{S}$ *is the vector* $P\vec{R}$ *determined by dropping a perpendicular from Q to the line PS determined by* $Proj_A{}^B$.

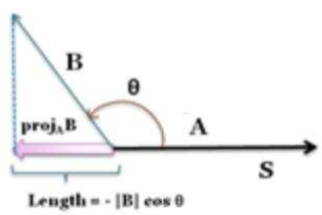
40. *If B represents a force vector then* $Proj_A{}^B$ *represents the effective force in the direction of A*

41. *If* θ *is the angle between B and A*

a. Case1: If θ is acute then $Proj_A^B$, has length $|B|\cos\theta = \frac{B\ A}{|A|}$ (Scalar component of B in the direction of A) and direction $\frac{A}{|A|}$



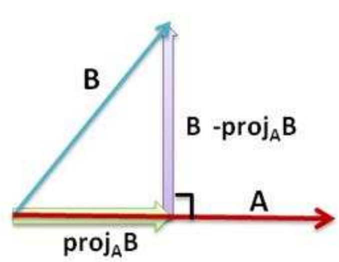
b. Case2: If θ is obtuse then $Proj_A^B$, has length $-|B|\cos\theta = -\frac{B\ A}{|A|}$ (Scalar component of B in the opposite direction of A) and direction $-\frac{A}{|A|}$



42. The vector projection of B onto A $\longrightarrow \Pr oj_A{}^B = \left(\frac{B\,.A}{|A|^2}\right) A$

43. The vector projection of A onto B $\longrightarrow \Pr oj_B{}^A = \left(\frac{A\,.B}{|B|^2}\right) B$

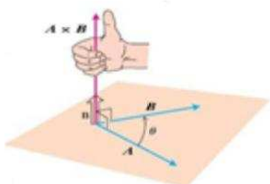
44. The vector B as a sum of a vector parallel to A and a vector perpendicular to A is $B = \underbrace{\Pr oj_A{}^B}_{\text{Parallel to } A} + \underbrace{\left(B - \Pr oj_A{}^B\right)}_{\text{Orthogonal to } A} = \underbrace{\left(\frac{B\,.A}{|A|^2}\right) A}_{\text{Parallel to } A} + \underbrace{\left(B - \left(\frac{B\,.A}{|A|^2}\right) A\right)}_{\text{Orthogonal to } A}$



45.The work done by a constant force F acting through a displacement d is $W = |F||d|\cos\theta$

46.Zero vector is the only vector which is orthogonal to every non zero vector.

47.Cross Product or Vector Product: The Cross product of two vectors A and B is denoted by A×B and is defined as follows $A\times B = (|A||B|\sin\theta)\vec{n}$. Where n is a unit vector perpendicular to plane containing the vectors A and B



48.The vector A×B is orthogonal to both A and B since it is scalar multiple of n

49.Non zero parallel vectors are parallel if and only if A×B=0

50.Properties of Cross Product:

If A,B and C are any three vectors and r,s are scalars then

a. (rA)×(sB)=(rs) (A×B) Scalar Distributive Law
b. A×(B+C)=A×B +A×C } Vector Distributive law
c. (B+C) ×A =B ×A + C ×A }
d. A×B=-(B×A)
e. 0×A=0

51.The product of i , j , k

$i\times j = k$; $j\times i = -k$

$j\times k = i$; $k\times j = -i$

$k\times i = j$; $i\times k = -j$

$i\times i = j\times j = k\times k = 0$

52.The area of the parallelogram determined by A and B is $|A||B|\sin\theta = |A\times B|$

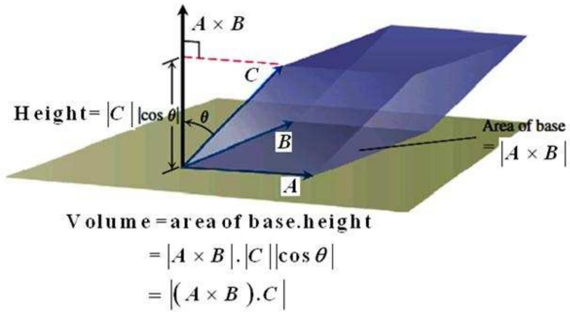
53.The area of the Triangle determined by A and B is $\frac{1}{2}|A||B|\sin\theta = \frac{1}{2}|A\times B|$

54.Magnitude of torque vector $|\tau| = |r||F|\sin\theta = |r\times F|$

55.Torque vector $\tau = (|r||F|\sin\theta)\vec{n} = r\times F$

56.If $A=a_1 i+b_1 j+c_1 k$ and $B=a_2 i+b_2 j+c_2 k$ then $A\times B = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

57.Volume of the pallelepiped: $V = |(A\times B)\ .\ C|$



58.Let P,Q,R be any three points in a space then

a. The vector perpendicular to the plane P,Q and R is $PQ\times PR$

b. The unit vector perpendicular to the plane P,Q and R is $\dfrac{PQ\times PR}{|PQ\times PR|}$

c. The area of the triangle formed by the vertices P,Q and R $=\frac{1}{2}|PQ\times PR|$

59.If $A=a_1 i+a_2 j+a_3 k$, $B=b_1 i+b_2 j+b_3 k$ and $C=c_1 i+c_2 j+c_3 k$ then the scalar triple product $[A\quad B\quad C]=(A\times B).C=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

60.The formula for area of the triangle with the vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $A = abs\frac{1}{2}\begin{vmatrix} x_1 & x_2 & 1 \\ y_1 & y_2 & 1 \\ z_1 & z_2 & 1 \end{vmatrix}$

61.Properties of Scalar Triple Product (Dot Product)

a. $[A\quad B\quad C]=[B\quad C\quad A]=[C\quad A\quad B]$

b. $(A\times B).C = A.(B\times C)$

62.If $(A\times B).C=0$ when

a. One of A ,B ,C is 0 or

b. A,B or B,C or C,A are collinear vectors or parallel vectors or

c. A,B,C are coplanar.

63.Vector triple Product: The product $(A\times B)\times C$ or $A\times(B\times C)$ is called the vector triple product of the three vectors are defined as

a. $(A\times B)\times C =(C.A)B-(C.B)A$

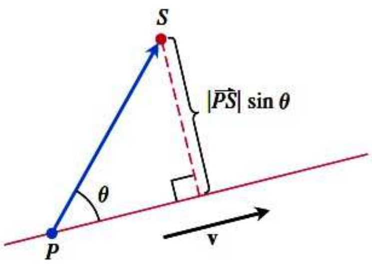
b. $A\times(B\times C) =(A.C)B-(A.B)C$

64.If B is perpendicular to both A and C ,then $(A\times B)\times C =A\times(B\times C)$

UNIT:V

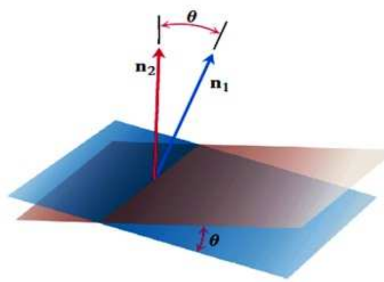
THREE DIMENSIONAL GEOMETRY

1. The vector equation for the line passing through the point $P_0(x_0,y_0,z_0)$ and parallel to the vetor V is $r(t)=r_0+tV$, $-\infty<t<\infty$
2. The standard parameterization for the line passing through the point $P_0(x_0,y_0,z_0)$ and parallel to the vector $V=v_1 i+v_2 j+v_3 k$ is $x=x_0+v_1t$ and $y=y_0+v_2t$ and $z=z_0+v_3t$ and $-\infty<t<\infty$
3. Distance from a point S to a line through P parallel to V is $d=\frac{|PS\times V|}{|V|}$



4. The vector equation for the plane passing through the point $P_0(x_0,y_0,z_0)$ and normal to the vetor $V=v_1 i+v_2 j+v_3 k$ is, $n\,.\,P_0\vec{P}=0$
5. The Component equation for the plane passing through the point $P_0(x_0,y_0,z_0)$ and normal to the vetor $V=A\,i+Bj+Ck$ is
   $\mathbf{A(x-x_0)+B(y-y_0)+C(z-z_0)=0}$ or $\mathbf{Ax+By+Cz=D}$ where $\mathbf{D=Ax_0+By_0+Cz_0}$
6. Two planes are parallel if and only if their normal n_1 and n_2 are parallel.
7. The line of intersection of two planes perpendicular to the both planes normals n_1 and n_2 and parallel to $n_1\times n_2$
8. If P is a point on a plane with normal n , then the distance d from any point S to the plane is the length of the projection of PS on n , i.e. $d=\left|\vec{PS}\,.\,\frac{n}{|n|}\right|$
9. The angle between two intersecting planes is defined to be the angle determined by the planes.

   If θ is the angle between the two planes then $\theta=\cos^{-1}\left(\frac{n_1\,.n_2}{|n_1||n_2|}\right)$



10.The cylinder is a surface that can be generated by a straight line along a given planar curve while holding the line parallel to some fixed line .the curve is called generating curve and the fixed line is called axis of the cylinder.

11.All cylindrical coordinates of the point (r , θ , z) is $(r,\theta,z)=(r,\theta+2n\pi,z)=(-r,\theta+(2n+1)\pi,z)$ here $n\in Z$

12.

Generating Curve	Cylinder parallel to	Axis of the cylinder	Cylinders equation
f(x,y)=c	Z-axis	Z-axis	f(x,y)=c
g(z,x)=c	Y-axis	Y-axis	g(z,x)=c
h(y,z)=c	X-axis	X-axis	h(y,z)=c

13. Quadric Surfaces:

a. Ellipsoids:

Standard Equation	Quadric Surface
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Ellipsoid
$\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{c^2}=1$ or $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{b^2}=1$ or $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{a^2}=1$ (i.e. either a=b or b=c or a=c)	Ellipsoid of revolution
$x^2+y^2+z^2=a^2$ (i.e. a=b=c)	Sphere

b. Paraboloids:

Standard Equation	Quadric Surface
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$	Elliptic Paraboloid
$\frac{x^2}{a^2}+\frac{y^2}{a^2}=\frac{z}{c}$(i.e.a=b)	Circular Paraboloid
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{z}{c}$	Hyperbolic paraboloid

c. Hyperboloids:

Standard Equation	Quadric Surface
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Hyperboloid of one sheet
$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=1$(i.e.a=b)	Hyperboloid of one sheet of revolution
$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Hyperboloid of two sheets.

d. Cones:

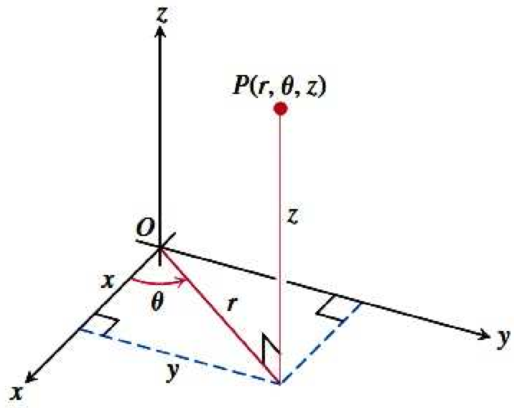
Standard Equation	Quadric Surface
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$	Elliptic Cone
$\frac{x^2}{a^2}+\frac{y^2}{a^2}=\frac{z^2}{c^2}$ (i.e.a=b)	Right Circular Cone

14. Cylindrical Coordinates:

Cylindrical Coordinates represents a point P in a space by ordered triples (r, θ, z) in which

a. r and θ are polar coordinates for the vertical projection of P on the xy-plane,

b. z is the rectangular vertical coordinate



15.The equations related to rectangular coordinates and cylindrical coordinates are

a. $x=r\cos\theta$, $y=r\sin\theta$, $z=z$

b. $r^2 = x^2 + y^2$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

16.

Equation	Two dimensional representation	Three dimensional representation
$r=0$	origin	Z=axis
$r=a$	circle	Circular cylinder parallel to Z-axis
$\theta=\theta_0$	Line passing through the origin making an angle θ_0 with positive X-axis	Plane containing the Z-axis and the line $\theta=\theta_0$
$r=2a\cos\theta$ and $r=2a\sin\theta$	Circle with radius a	Circular cylinders with radius a

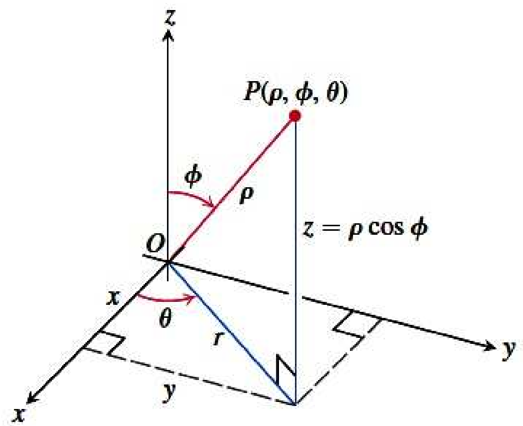
17.Spherical Coordinates:

Spherical Coordinates represent a point P in a space by ordered triples (ρ, φ, θ) in which

a. ρ is the distance from P to origin

b. φ is the angle $|\vec{OP}|$ makes with positive Z-axis $(0 \le \varphi \le \pi)$

c. θ is the angle from cylindrical coordinates.



18.If (ρ , φ , θ) are spherical coordinates of a point in a space. we find Cartesian and cylindrical coordinates by using the formula

Rectangular coordinates	$x=\rho \sin\varphi \cos\theta$, $y= \rho \sin\varphi \sin\theta$ $z=\rho\cos\varphi$
Cylindrical coordinates	$r=\rho\sin\varphi$, $\theta= \theta$, $z=\rho\cos\varphi$

19.If (r , θ, z) are cylindrical coordinates of a point in a space. we find Cartesian and Spherical coordinates by using the formula

Rectangular coordinates	$x=r \cos\theta$, $y= r \sin\theta$ $z=z$
Spherical coordinates	$\rho=\sqrt{r^2+z^2}$ and $\varphi=\tan^{-1}\left(\frac{r}{z}\right)$ and $\theta=\theta$

20.If (x , y, z) are Cartesian coordinates of a point in a space. we find Cylindrical and Spherical coordinates by using the formula

Cylindrical coordinates	$r=\sqrt{x^2+y^2}$ and $\theta=\tan^{-1}\left(\frac{y}{x}\right)$ and $z=z$
Spherical coordinates	$\rho=\sqrt{x^2+y^2+z^2}$ and $\varphi=\tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$ and $\theta=\tan^{-1}\left(\frac{y}{x}\right)$

21.

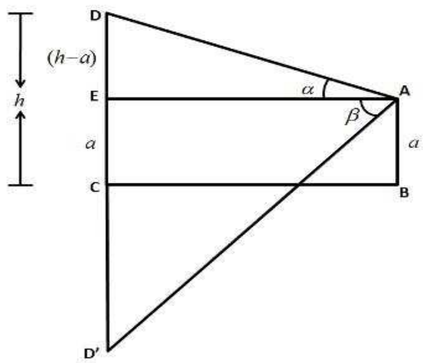
Equation	*Representation*
$\rho=a$	*Sphere of radius **a** centered at origin*
$\varphi=\varphi_0 \quad (\varphi_0<\pi/2)$	*Cone opening up from the origin making an angle φ_0 radius with positive Z-axis*
$\varphi=\pi/2$	*Represents a xy-plane*
$\varphi=\varphi_0 \quad (\varphi_0>\pi/2)$	*Cone opening down from the origin making an angle φ_0 radius with positive Z-axis*
$\varphi=0$	*Represents a positive Z-axis*
$\varphi=\pi$	*Represents a negative Z-axis*
$\theta=\theta_0$	*Half plane that contain the Z-axis and makes an θ_0 with positive X-axis*

22. *The equation* $z=\sqrt{x^2+y^2}$ *represents a right circular cone whose spherical equation is* $\varphi=\pi/4$.

UNIT=VI

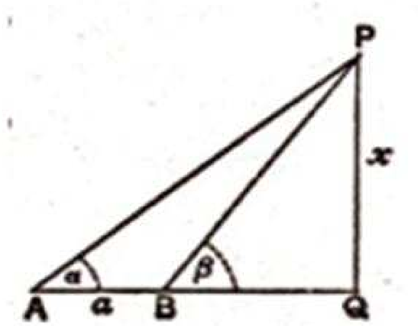
TRIGONOMETRY

1. *From the top of a tree on the bank of the lake, an aero plane in the sky makes an angle of elevation α and an angle of depression β. If the height of the tree from the surface is "a" and the height of the aero plane is* $h = \dfrac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$

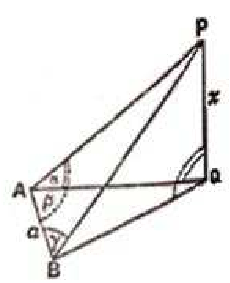


2. *The height of an inaccessible tower :-*

   a. $h = \dfrac{a\sin\alpha sn\beta}{\sin(\beta-\alpha)}$



   b. $h = \dfrac{a\sin\alpha sn\gamma}{\sin(\beta+\gamma)}$



3. *Area of the triangle* $\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$
4. *Area of the triangle(Heron's formula)* $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ *where* $s = \frac{a+b+c}{2}$
5. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ *here R is radius of the circum circle*
6. *Radius of the incircle:*
   a. $r = \frac{\Delta}{s}$
   b. $r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$
   c. $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
7. *Radius of the exterior circle:*
   a. $r_1 = \frac{\Delta}{s-a}$ *and* $r_2 = \frac{\Delta}{s-b}$ *and* $r_3 = \frac{\Delta}{s-c}$
   b. $r_1 = s\tan\frac{A}{2}$ *and* $r_2 = s\tan\frac{B}{2}$ *and* $r_3 = s\tan\frac{C}{2}$
   c.
   $$r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$
   $$r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$$
   $$r_3 = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$
8. *Some Important Results:*
   a. $abc = 4R\Delta$
   b. $rr_1r_2r_3 = \Delta^2$
   c. $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$
   d. $r_1r_2 + r_2r_3 + r_3r_1 = s^2$
   e. $r_1 + r_2 + r_3 - r = 4R$
   f. $r + r_2 + r_3 - r_1 = 4R\cos A$
   g. $r + r_1 + r_3 - r_2 = 4R\cos B$
   h. $r + r_1 + r_2 - r_3 = 4R\cos C$

   (f–h) *In this case triangle is right angle triangle*

9. *Sum of the sine angles in A.P.*

$$\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+---+\sin(\alpha+(n-1)\beta)=\frac{\sin\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$$

10. *Sum of the cosine angles in A.P.*

$$\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+---+\cos(\alpha+(n-1)\beta)=\frac{\cos\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$$

11. $\sin\alpha+\sin\left(\alpha+\frac{2\pi}{n}\right)+\sin\left(\alpha+\frac{4\pi}{n}\right)+----+\sin\left(\alpha+(n-1)\frac{2\pi}{n}\right)=0$

12. $\cos\alpha+\cos\left(\alpha+\frac{2\pi}{n}\right)+\cos\left(\alpha+\frac{4\pi}{n}\right)+----+\cos\left(\alpha+(n-1)\frac{2\pi}{n}\right)=0$

13. $\frac{\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+---+\sin(\alpha+(n-1)\beta)}{\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+---+\cos(\alpha+(n-1)\beta)}=\tan\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}$

14. $\frac{\sin\alpha-\sin(\alpha+\beta)+\sin(\alpha+2\beta)----+(-1)^{n-1}\sin(\alpha+(n-1)\beta)}{\cos\alpha-\cos(\alpha+\beta)+\cos(\alpha+2\beta)----+(-1)^{n-1}\cos(\alpha+(n-1)\beta)}=\tan\left\{\alpha+\left(\frac{n-1}{2}\right)(\pi+\beta)\right\}$

15. $\frac{\sin\alpha+\sin2\alpha+\sin3\alpha+---+\sin n\alpha}{\cos\alpha+\cos2\alpha+\cos3\alpha+----+\cos n\alpha}=\tan\left(\frac{n+1}{2}\right)\alpha$

16. $\frac{\sin\alpha+\sin3\alpha+\sin5\alpha+----+\sin(2n-1)\alpha}{\cos\alpha+\cos3\alpha+\cos5\alpha+----+\cos(2n-1)\alpha}=\tan n\alpha$

17. *Demoivre's Theorem for rational index:*

If θ is any real number and n is any integer ,then

$(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

18. *Let Z=r(cosφ+i sinφ) be the n^{th} root of complex number a=ρ(sinθ+i sinθ)*

then the n^{th} roots are $z=\sqrt[n]{r}\left(\cos\left(\frac{\varphi+2k\pi}{n}\right)+i\sin\left(\frac{\varphi+2k\pi}{n}\right)\right)\quad k=0,1,2,---(n-1)$

19. *If* $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ *then the following are true*

a. $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$

b. $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$

c. $\cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)=0$

d. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}$ *and* $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=\frac{3}{2}$

20. $\sin(\alpha_1+\alpha_2+\alpha_3+------)=\cos\alpha_1\cos\alpha_2\alpha_3------[s_1-s_3+s_5-s_7+--------]$

21. $\cos(\alpha_1+\alpha_2+\alpha_3+------)=\cos\alpha_1\cos\alpha_2\alpha_3------[1-s_2+s_4-s_6+--------]$

22. $\tan(\alpha_1+\alpha_2+\alpha_3+------)=\dfrac{s_1-s_3+s_5-s_7+--------}{1-s_2+s_4-s_6+--------}$

23. $\cos n\theta=\cos^n\theta-n_{c_2}\cos^{n-2}\theta\sin^2\theta+n_{c_4}\cos^{n-4}\theta\sin^4\theta------------$

24. $\sin n\theta=n_{c_1}\cos^{n-1}\theta\sin\theta-n_{c_3}\cos^{n-3}\theta\sin^3\theta+n_{c_5}\cos^{n-5}\theta\sin^5\theta------------$

25. $\tan n\theta=\dfrac{n_{c_1}\tan\theta-n_{c_3}\tan^3\theta+n_{c_5}\tan^5\theta------------}{1-n_{c_2}\tan^2\theta+n_{c_4}\tan^4\theta----------------}$

26. $\sin\theta=\theta-\dfrac{\theta^3}{3!}+\dfrac{\theta^5}{5!}-\dfrac{\theta^7}{7!}+-----------$

27. $\cos\theta=1-\dfrac{\theta^2}{2!}+\dfrac{\theta^4}{4!}-\dfrac{\theta^6}{6!}+-----------$

28. *Useful identities in hyperbolic functions*

IDENTITIES

(1) $\cosh^2 x-\sinh^2 x=1,\ \forall x\in R$

(2) $1-\tanh^2 x=\operatorname{sech}^2 x,\ \forall x\in R$

(3) $\coth^2 x-1=\operatorname{cosech}^2 x,\ \forall x\in R-\{0\}$

(4) $\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y,\ \forall x,y\in$ □

(5) $\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y,\ \forall x,y\in$ □

(6) $\tanh(x+y)=\dfrac{\tanh x+\tanh y}{1+\tanh x\tanh y}$

(7) $\sinh(x-y)=\sinh x\cosh y-\cosh x\sinh y$

(8) $\cosh(x-y)=\cosh x\cosh y-\sinh x\sinh y,$

(9) $\tanh(x-y)=\dfrac{\tanh x-\tanh y}{1-\tanh x\tanh y}$

(10) $\sinh 2x=2\sinh x\cosh x=\dfrac{2\tanh x}{1-\tanh^2 x}$

(11) $\cosh 2x=\cosh^2 x+\sinh^2 x=2\cosh^2 x-1$
$=1+2\sinh^2 x=\dfrac{1+\tanh^2 x}{1-\tanh^2 x}$

HYPERBOLIC FUNCTIONS

29.Inverse Hyperbolic functions:

Inverse Hyperbolic Functions

(12) $\sinh^{-1} x = \ln\left(x+\sqrt{x^2+1}\right), \quad x \in R$

(13) $\cosh^{-1} x = \ln\left(x+\sqrt{x^2-1}\right), x \in [1,\infty]$

(14) $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad x \in (-1,1)$

(15) $\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) \quad |x|<1$

(16) $\text{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right), \quad x \in (0,1)$

(17) (a) $\text{csch}^{-1} x = \ln\left(\frac{1-\sqrt{1+x^2}}{x}\right), \quad x \in (-\infty,0)$

(b) $\text{csch}^{-1} x = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right),$

30.If $x = \ln\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$ *then coshx=secθ and sinhx=tanθ*

31. $\sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \tanh^{-1} x$

32.cos ix=cos hx and sin ix=i sin hx and tan ix=i tan hx

UNIT-VII

LINEAR PROGRAMMING

1. *Optimization problem:*
   The problem which seeks to maximum or minimize a linear function (say in two variables x and y) subject to certain constraints is called Optimization problem
2. *Linear programming problem:*
   A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum value) of a linear function (called ***objective function****) and satisfy a set of linear inequalities (called* ***linear constraints****)*
3. *Feasible region:*
   The common region determined by all the constraints (including the non negative constraints) of a LPP is called the feasible region
4. *Feasible solution:*
   Any solution that satisfies all constraints of a LPP is called Feasible solution (or) points within and the boundary of the feasible region is called Feasible solution
5. *Unfeasible solution:*
   The points outside the feasible region are called the unfeasible solution
6. *Optimal Feasible solution:*
   Any Point in the feasible region that gives the optimal value (maximum or minimum value) of the objective function is called the optimal solution.
7. *Corner Point:*
   The intersection of the two boundary lines of a feasible region R is called the Corner Point of the feasible region R.

UNIT-VIII

PROBABILITY

1. The set of all possible outcomes of an experiment is called the sample space and it is denoted by S
2. The outcomes of an experiment are called the elementary event or sample event or sample point.
3. The subset of a sample space is called an event.
4. Two events A and B are said to be mutually exclusive if $A \cap B = \phi$
5. Two events A and B are said to be exhaustive if $A \cup B = S$
6. If a coin is tossed n times the number of possible outcomes are 2^n
7. If a die is tossed n times the number of possible outcomes are 6^n
8. The occurrence of an event resulting from such an experiment is evaluated by set of real numbers called probabilities.
9. To every point in a sample space assign a probability such that sum of all probabilities is one.
10. For the points outside the sample space assign a probability of zero.
11. If an experiment can result in any one of the N different equally likely outcomes and if exactly n of these outcomes correspond to an event A, then the probability of the event A is $P(A) = \frac{n(A)}{n(S)} = \frac{n}{N}$
12. $0 \le P(A) \le 1$
13. If A and B are two events ,then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
14. If A and B are two mutually exclusive events ,then $P(A \cup B) = P(A) + P(B)$
15. If A_1, A_2, A_3,----------- A_n are mutually exclusive events, then $P(A_1 \cup A_2 \cup A_3 \cup ----------- \cup A_n) = P(A_1) + P(A_2) + P(A_3) + --- + P(A_n)$
16. Partition of S: the collection of events A_1, A_2, A_3,----------- A_n of a sample space are said to be partition of S if
    a. A_1, A_2, A_3,----------- A_n are mutually exclusive
    b. $A_1 \cup A_2 \cup A_3 \cup ----- \cup A_n = S$
17. If A_1, A_2, A_3,----------- A_n is a partition of S then $P(A_1 \cup A_2 \cup A_3 \cup ---- \cup A_n) = 1$
18. If A, B, and C are any three events of a sample space, then $P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A) - P(A \cap B \cap C)$

19.*If A is any event then* $P(A^1)=1-P(A)$

20.*Conditional Probability: If A and B are two events in a sample space S and P(A)≠0 then the probability of B after the event A has occurred is called the conditional probability of the event B given A and is denoted by P(B/A) and* $P(B/A)=\frac{P(B\cap A)}{P(A)}$

21.*Independent events: Two events A and B are said to be Independent events if P(B/A)=P(B) or P(A/B)=P(A)*

22.*If in an experiment the events A and B can both occur ,then*
$P(A\cap B)=P(A)P(B/A)$

23.*Two events A and B are said to be Independent events if*

a. $P(A\cap B)=P(A)P(B)$

b. $P(A\cup B)=P(A)+P(B)-P(A)P(B)$

c. *The events* A^1 *and* B^1 *are also independent*

24.*Theorem of total probability:*

If B_1, B_2, B_3,----------- B_k *constitute a partition of the sample space S such that* $P(B_i)\neq 0$,*for i=1,2,3,4,-------,k then for any event A of S*

$$P(A)=\sum_{i=1}^{k}P(B_i\cap A)=\sum_{i=1}^{k}P(B_i)P(B_i\cap A)$$

25.*Baye's Rule:*

If B_1, B_2, B_3,----------- B_k *constitute a partition of the sample space S such that* $P(B_i)\neq 0$,*for i=1,2,3,4,-------,k then for any event A of S*

$$P(B_r/A)=\frac{P(B_r\cap A)}{P(A)}=\frac{P(B_r)P(A/B_r)}{\sum_{i=1}^{k}P(B_i)P(B_i\cap A)}$$

26.*Random Variable:*

A random variable X on a sample space S is a function $X:S\to R$, *where R is the set of real numbers , which assign a real number X(s) to each element 's' in S.*

27.*Countable set or Denumerable Set:*

A set E is said to be countable if E is either finite or there exists a 1-1 correspondence between E and N.

28.*Discrete Sample Space:*

A sample space S is said to be discrete if it is countable.

29. Continuous Sample Space:

If a sample space contains an infinite number of infinite number of possibilities equal to the number of points on a line segment, it is called a continuous sample space.

30. Discrete random variable:

A random variable is called a discrete random variable if its set of possible outcomes is countable

A ***discrete*** *random variable can take on only specific, isolated numerical values, like the outcome of a roll of a die, or the number of dollars in a randomly chosen bank account. Discrete random variables that can take on only finitely many values (like the outcome of a roll of a die) are called* ***finite*** *random variables. Discrete random variables that can take on an unlimited number of values (like the number of stars estimated to be in the universe) are* ***infinite discrete*** *random variables.*

Examples:

a. Flip a coin three times; X = the total number of heads.
b. Select a mutual fund; X = the number of companies in the fund portfolio.
c. Throw two dice over and over until you roll a double six; X = the number of throws.
d. Take a true-false test with 100 questions; X = the number of questions you answered corectly.
e. Invest $10,000 in stocks; X = the value, to the nearest $1, of your investment after a year.

31. Continuous random variable:

A random variable is said to be continuous random variable if it can take on any value between two specified values. i.e., if it can take on values on a countable scale.

A ***continuous*** *random variable, on the other hand, can take on any values within a continuous range or an interval, like the temperature in Central Park, or the height of an athlete in centimeters.*

Examples :

a. Measure the length of an object; X = its length in centimeters.

b. Select a group of 50 people at random;
X = the exact average height (in m) of the group.

32. Probability Function:

The set of all ordered pairs (x , f(x)) is called a probability function or probability mass function or probability distribution of the discrete random variable X , if each possible outcome x

a. $f(x) \geq 0$
b. $\sum_{x} f(x) = 1$
c. $P(X = x) = f(x)$

33. The cumulative distributive function F(x) of a discrete random variable X with probability function f(x) is defined by
$F(x) = P(X \leq x) = \sum_{t \leq x} f(t)$, for $-\infty < x < \infty$

34. $P(X \leq x_k) = P(x_1) + P(x_2) + P(x_3) + --------- + P(x_k)$

35. $P(X < x_k) = P(x_1) + P(x_2) + P(x_3) + --------- + P(x_{k-1})$

36. $P(X > x_k) = 1 - P(X \leq x_k) = 1 - [P(x_1) + P(x_2) + P(x_3) + --------- + P(x_k)]$

37. $P(X \geq x_k) + P(X < x_k) = 1$

The essence of mathematics is not to make simple things complicated, but to make complicated things simple.

ALL THE BEST